

Title: Material Swarm Articulations

Subtitle: New View, Reciprocal Frame Canopy



Authors: Evangelos Pantazis, David J. Gerber

Abstract: Material Swarm Articulations is an experiment in developing a multi-objective optimization system that incorporates bottom up approaches for informing architectural design. The paper presents an initial built project that demonstrates the combination of a structural form finding method, with an agent based design system through the digital fabrication processes. The objective of this research is to develop a workflow combined with material and construction constraints that has the potential to increase performance objectives while enabling geometric complexity and design driven articulation of a traditional tectonic system. The emphasis of the research at this stage is to take advantage of material properties and assembly methods applied to a digital design and simulation workflow that enables emergent patterns to influence the performance of the space. The paper illustrates the research through a prototype of a self-standing canopy structure in 1:1 scale. It presents results of the form finding, generative patterning, digital fabrication affordances and sets and agenda for next steps in the use of multi-agent systems for design purposes.

Keywords: *Computational design; agent-based system; digital fabrication; parametric design; reciprocal frames; form finding; multi-objective optimization, multi-agent systems for design*

Introduction and Research Objectives

In contemporary design discourse and practice the rapid development and availability of computational design tools are amplifying both conceptual and technical capacity for manipulating complex geometrical configurations and introducing informed and articulated pioneering design possibilities. (Gerber and Lin, 2013, Tsiliakos, 2012). Moreover computerized fabrication technologies have enabled the generation of unique mass customized parameterized parts for almost the same cost of standardized production issuing in the post fordist paradigm (Scheurer et al., 2005). As a result, a great number of architectural projects worldwide are being realized partly following a “file-to-factory” pattern, as the progress and availability of fabrication technologies has allowed the manufacturing of complex geometries to a large extent. As these new technologies in conjunction with contemporary social phenomena suggest a more participatory design model that supports customization and the sharing of knowledge and tools, the notion of the vernacular in architecture, with local common rules and cultural values, is ripe for re-evaluation as to its design possibilities (Morel, 2006). The implication of the post fordist

paradigm in this context is to enable a more diverse set of design opportunities, styles and palettes while furthering the projects towards meeting global challenges for performance criteria inclusive of cost, energy, comfort, as well as social suitability and purpose.

While the introduction of digital fabrication in building industry is rapidly narrowing the tolerances between represented and realized form (Scheurer, 2005), recent investigations are looking into performative approaches of form generation that for instance seek to achieve a continuous digital chain from design exploration to fabrication which not only rationalizes structure but also evaluates the materialization process that leads to fabrication. (Hensel et al., 2010) Driven by increasing computational capacity and availability of data and simulation, some contemporary research is oriented towards integrating rationalization and optimization methods during the first steps of computational form finding. (Gerber, 2007) In terms of our use of computation, our project’s form can be defined as an interaction between internal components and external forces (Kwinter and

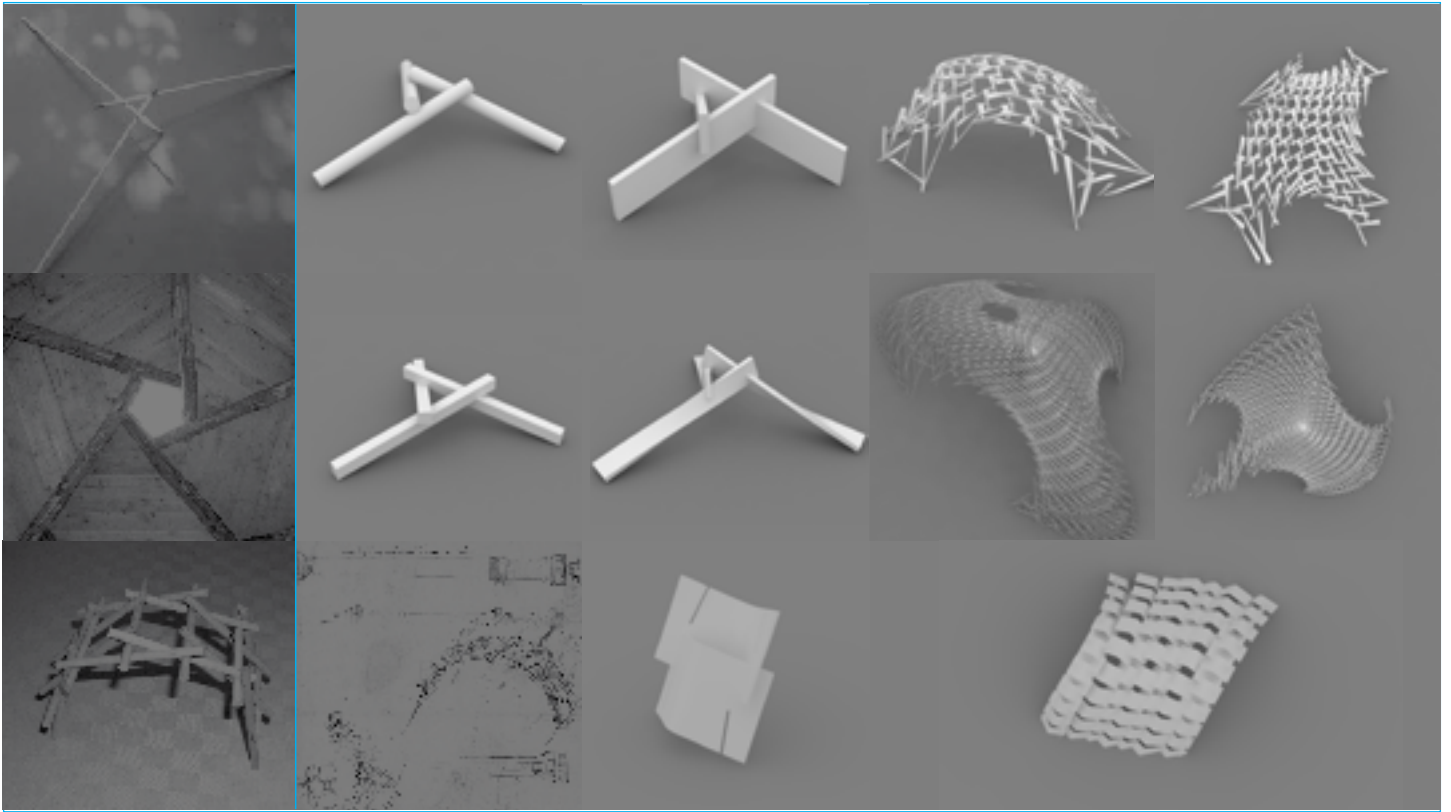


Figure 1: Studies on reciprocal frames units and aggregations with varying amount of elements and geometry

Davidson, 2008). These internal components are described as separate design drivers, which in turn, are synthesized into an integrated computational design tool, which take into account external forces, i.e. load cases and environmental parameters. These integrated design and performance drivers are described as agents with different motivations (i.e. material, social) that interact with each other within an environment, and exchange information that ultimately increase the complexity of the system as a whole (Baharlou and Menges, 2013). In this context, this work introduces a methodological framework that explores the applicability of form finding techniques and multi agent systems as a generative bottom up strategy for material and locally responsive design exploration coupled with traditional top-down design strategies and analysis tools. A primary objective at this stage of the research is to investigate how the integration of generative systems with constraints relating to structural systems, environmental criteria, and material and tectonic properties can lead us to enriched and complex design outcomes, a complexly curved canopy comprised of a novel reciprocal curvilinear frame. A second objective of the research is to reconsider architectural design strategies, by revising a vernacular building system that of the reciprocal frame, and combining it with an agent based model for informing the design process. A final objective is to test and validate the design methodology

that bridges the digital to the physical while addressing real world architectural constraints of time, cost, and construction. Advances in building technology and structural systems are traditionally an indicator of technological progress for a particular locality and culture. (Lin and Gerber, 2014) In the context of the technological leap and mass customization paradigm driven by access to computation we also consider it essential that the re-examination of traditional design, tectonic and structural methodologies be addressed. While the focus of the work is first to prove out the methodological approach an underlying agenda is also to ensure the approach is supportive of cultural design diversity.

Background and Context of Reciprocal Frames

The principle of reciprocity in structural design and construction i.e. the use of load bearing elements to compose a spatial configuration wherein they are mutually supported one another has been known since the antiquity (Pizzigoni, 2010). Etymologically reciprocity derives from the 'Latin' reciprocus, which is composed by the two parts 'recus' meaning backwards and 'procus' translated as forwards. The word, reciprocity, implies the practice of exchanging things with others for mutual benefits. Such a definition emphasizes the obliged stressed return of a certain action.

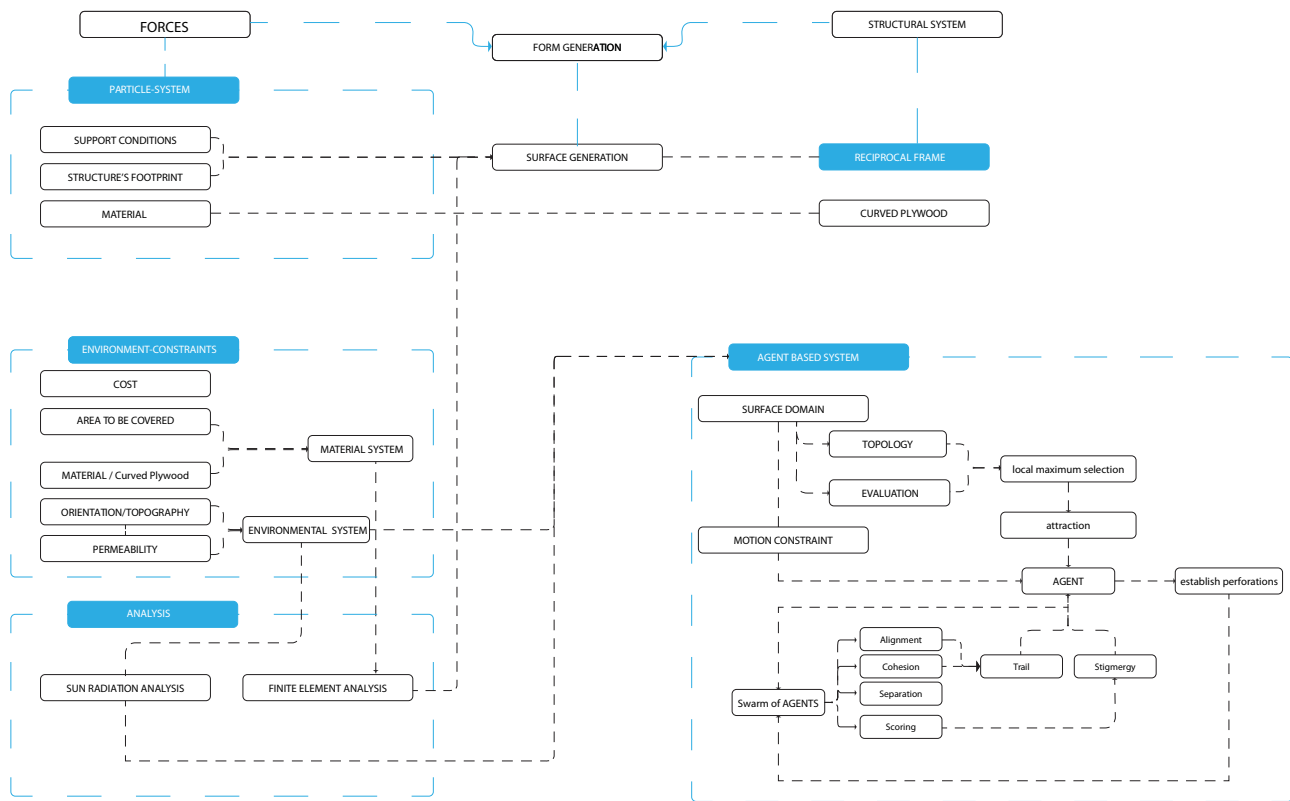


Figure 2: Workflow diagram

The development of reciprocal frames has not had a linear history and the evidences of its knowledge and application around the world seems to be unrelated to one another. However a common point in the use of this system is the use of timber as constructive material in both Occidental and Oriental culture. Thus it is worth noting that it was more of a practical and construction issue in Europe, for development of planar spanning configurations, while in Asia, for more ceremonial realization of three-dimensional structures.

The first reciprocal frame structures are traced back to Chinese and Japanese religious architecture in the 12th century, seen in the wood constructed roof support systems of the 'mandala' roof. In Europe, the concept of spanning distances longer than the length of the available timber beams was the main reason for the use and development of the reciprocity principle (Larsen, 2008). During the 13th century Villard de Honnecourt conceptualized in his sketches roof support structures that were based on this principle. Later in the 16th century Leonardo Da vinci, who laid the foundation for a scientific study of reciprocal structures, explored at least five different spatial configurations based on the principle of reciprocity, experimenting upon regular and non-regular 2D and 3D geometrical configurations. Sebastiano Serlio addressed the problem of planar roof construction with short beams in his book on architecture dated

to 1556 . A comparable structure system made of reciprocally supporting bar-shaped elements is the 'Zollinger system' which is mainly used in timber roof construction, where Friedrich Zollinger obtained a patent for it in 1923 (Kohlhammer and Kotnik, 2011). It has been through the development of sophisticated timber products—such as glulam trusses and plywood—that produce long spanning structural elements through adhesive technology that have led to the replacement and lack of further development of reciprocal frames and similar structure systems. Currently, the principle of reciprocity continues to stimulate the interest of designers and researchers and it has again become a topic for academic research. (Thomas Kohlhammer, 2010). Architectural applications include the Mill Creek Public Housing by Louis Kahn (1952-53), the Bunraku Puppet Theater by Seiwa, Kazuhiro Ishii (1994) and the Pompidou Metz museum by Shigeru Ban (2008) to name a few can be found around the world. Moreover, a set of experimental works related to structural, geometrical and constructive issues of 'reciprocal' structures are appearing, such as the Forest Park structure by Shigeru Bahn and ARUP AGU, the Serpentine pavilion by Cecil Balmond and Alvaro Siza (2005), the H-edge pavilion by Cecil Balmond and students from Penn University (Pugnale et al., 2011) and the research pavilions by at EPFL Lausanne (Nabaei and Weinand, 2011). In those projects the reciprocal principle has been

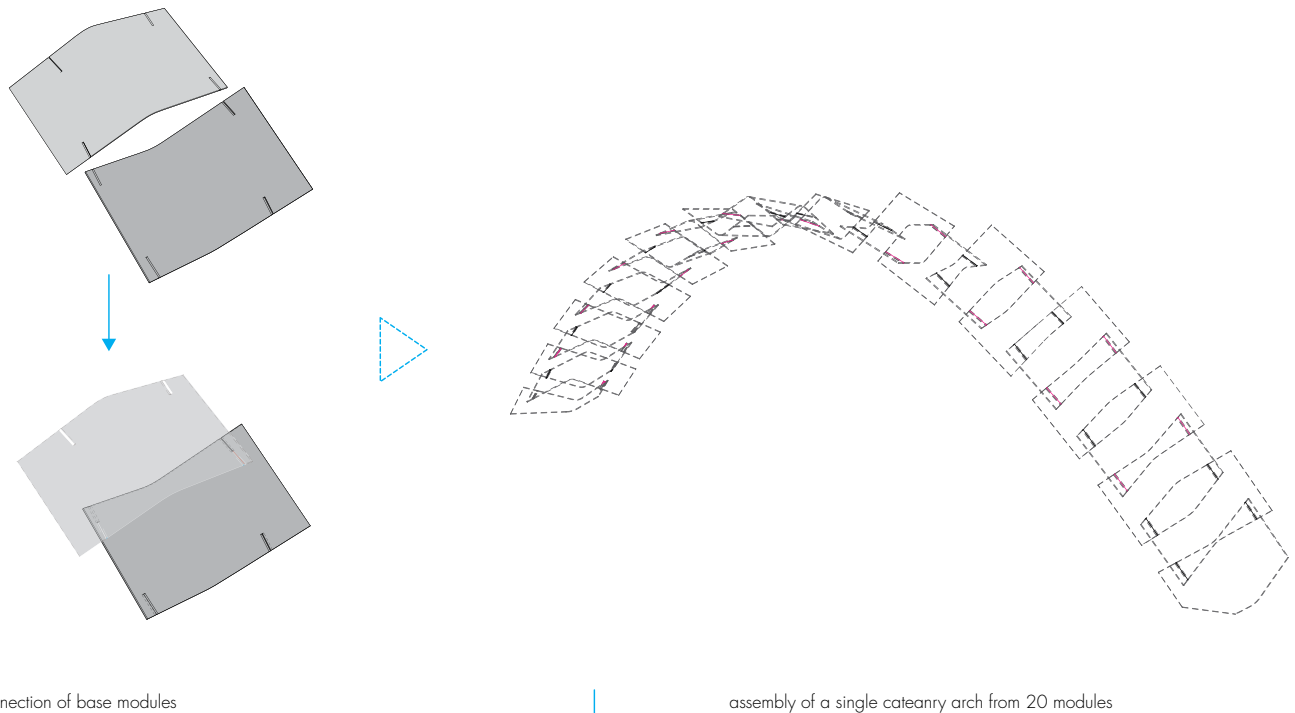


Figure 3: Diagram of the interlocking system of the basic components

explored by using different materials, element sections, joints and planar or 3-dimensional configurations providing the fundamental evidence that adaptations of this typology should still be further investigated in for a diversity of architectural styles, patterns, performance characteristics and finally local sensibilities. Based on a specific type of reciprocal frame, we re-examine the applicability of such a vernacular structural system by analyzing its functional and material behavior. The research models a design system that fosters design exploration by incorporating issues of recyclability and material efficiency coupled with the design and spatial comfort performance objectives. This is partially achieved by implementing digital fabrication techniques but also through the incorporation of agent based design technologies enabling emergent and intrinsic performance.

Reciprocal Frames Defined

A reciprocal frame is a structural system, formed by a number of short bars that are connected using friction only. Most importantly the reciprocal frame can span many times the length of the individual bars. Reciprocal refers to the fact that such structures are composed of a number elements (referred to also as “short beams”) that structurally interact through simple support binding in order to create more complex structures of dimensions much greater than the

single elements from which they are composed. The application of the reciprocity principle requires: a) the presence of at least two elements allowing the generation of forced interactions; b) that each element of the assembly must support and be supported by another one; and c) that every supported element must meet its support along the span and never at the vertices in order to avoid the generation of a space grid with pin joints. The space structures that conform with the above requirements are called reciprocal and are constituted of at least two interlaced linear elements, where the final form is relative to a basic component type, its material as well as the connection technology. The components can be identical or non-identical but should follow a specific global tessellation pattern. The joining of the components at the node points can generally be carried out without mechanical connections, solely by pressure and friction (Figure 1). To support the frictional forces simple connection techniques such as tying together or notching of the elements can be used. In fact, in the context of wood processing techniques it’s recognized that the complexity of the connection technology, becomes an important feature in order to distinguish different structural propositions both from financial and structural aspects (Nabaei and Weinand, 2011). From a structural point of view, each individual element in the system works as a single beam. This beam lies at each of its edges either on another component or if it forms

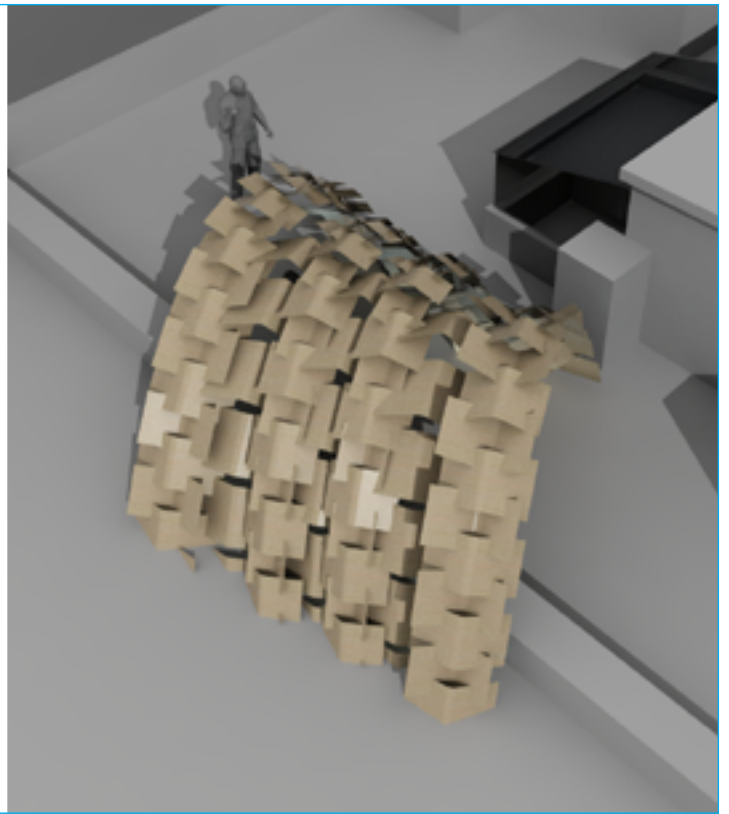


Figure 4: Renderings of the pavilion proposal on site

the edge of the structure, on the supports of the entire system. Each element bears the supporting force of one of the neighboring elements and optional dead loads or live loads. The interest of such structural system lays in the fact that the “global” form is determined by the “local” condition of the building elements.

Structural Form finding

The term, structural form finding is commonly used to recall a morphogenetic process used to find the relaxed form of grid-shells under a certain boundary condition and load conditions. Many techniques have been developed both in the academia as well as in practice in order to determine geometries that work in pure tension or compression under their self-weight. What is common between various form finding techniques is the fundamental property that the final form is the direct result of the force equilibrium and is influenced intensively by the materiality and the boundary condition applied (Nabaei and Weinand, 2011). Many designers historically have experimented with hanging models and other physical methods for finding efficient structural form acting in pure tension or compression. Antoni Gaudi employed hanging models to solve structural issues for projects like Casa Mila Pedrera, while Heinz Isler and Frei Otto have done extensive work developing highly accurate physical experiments for exploring

structural forms (Kilian and Ochsendorf, 2005). Alternatively such forms are called funicular, which etymologically derives from the Latin word ‘funiculus’, meaning thin cord. This refers to the shape taken by a thin cord acting in pure tension under a given set of loads. A new generation of researchers within the field of design and computing are developing computational tools implementing the same principles in digital environments (Van Mele et al., 2013). They provide tools that often employ particle spring systems and aim to educate designers as to the effects of forces on the form of structure as well as provide an interactive form finding environment that was previously restricted to physical models.

Particle-spring systems are based on lumped masses, called particles, which are connected by linear elastic springs. Each particle in the system has a position, a velocity and variable mass, as well as a summarized vector for all forces acting on it (gravity, loads etc). These forces can be calculated using mass less connectors between particles, called springs (Kilian and Ochsendorf, 2005). The magnitude of these forces is based on the spring’s offset from its rest length, and thus supports can be added by restraining the displacement of specific particles. The system initiates in a non-equilibrium state, and particles move until they reach their equilibrium positions for a given set of parameters. In this research we investigate the possibilities of such a system

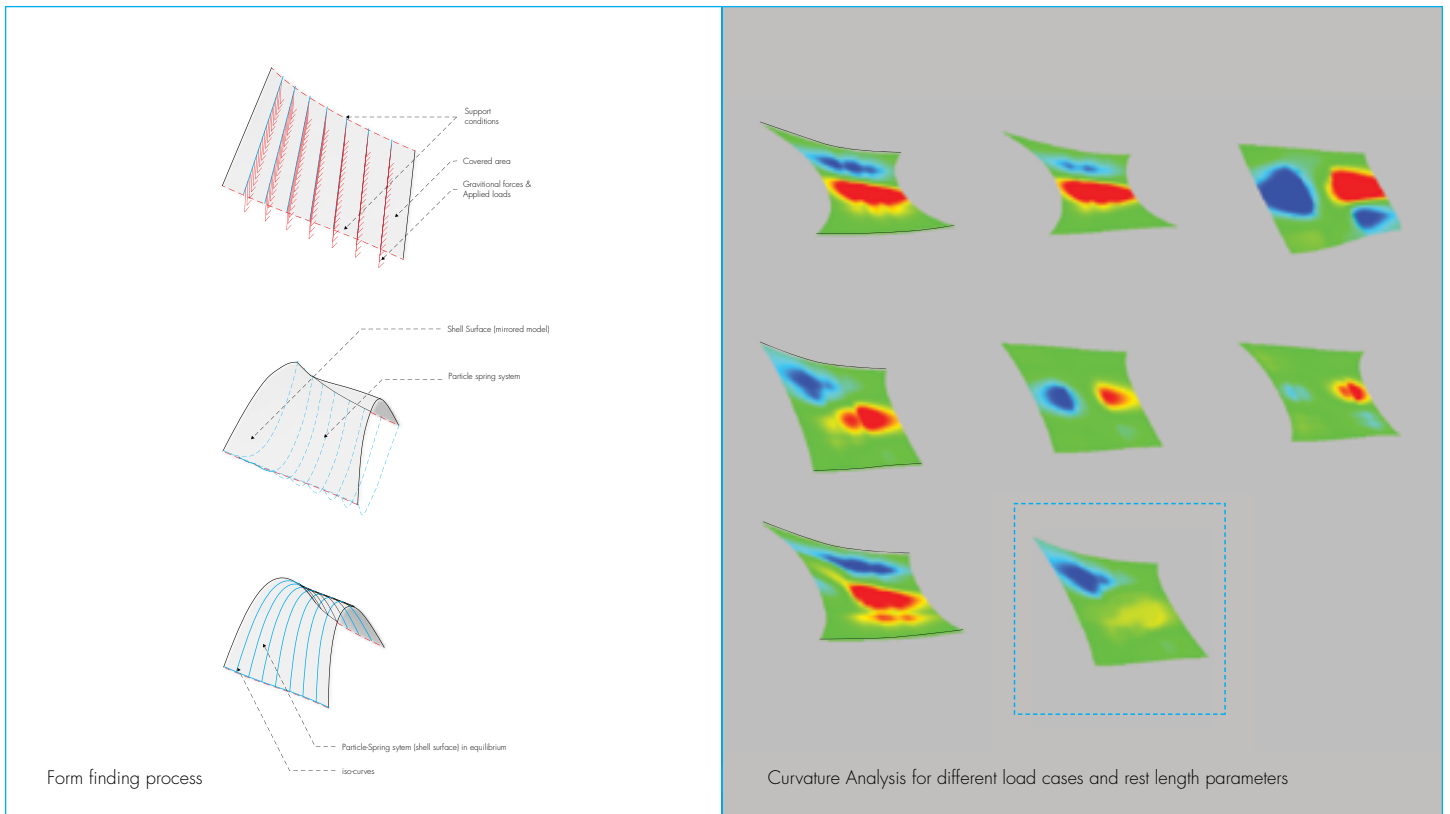


Figure 5: Form finding principles and curvature analysis diagrams

for the introduction of structural evaluation environments into the design process from the early stage of our design workflow. (Figure 2). In particular we explore a self-standing canopy shell structure by approximating a hanging model of networked catenary arches. For that we developed a custom definition and algorithm using a particle spring system through the interface of an open source plugin (Kangaroo) that operates within a commercial 3d software package (Rhinceros-Grasshopper).

Agent Based Design Systems

An agent-based design system consists of large number of agents that follows simple local rules and interacts within an environment (Gilbert, 2008). Critically, the research addresses the fact that functioning in a similar fashion that goes from local relations to emergent global phenomena the reciprocal principle asks for the implementation of a bottom up design approach. Such an approach maintains a generative computational protocol to generate varying design possibilities, while maintaining specific material constraints or local limitations. While most designers engage with the information from simulations as a form of validation or as an aid in the decision making process, there are numerous precedents that have used the information from simulation as a platform and a driver in the generative process (Miranda and Coates, 2000, Ireland, 2009). As

a first step towards such a design paradigm we developed an agent-based system for introducing a perforation pattern that is informed by an environmental analysis. Specifically, a flocking algorithm is combined with data resulting from a sun radiation analysis in order to investigate the permeability of the selected structural system. As a first step basic motion behavior mechanisms are introduced to agents that spawn at specified points on the surface of the components. Further behaviors (attraction, repulsion) are triggered by values provided from environmental analysis, specifically sun radiation measured as perceived energy (kWh) / area (sqm). The basic motion behavior mechanisms are introduced according to the definition of Reynolds who categorizes them in three layers; action, selection, steering and locomotion (Reynolds, 1987). At issue for our research are the mechanisms of these generative agent-based systems how they are bound to material properties, fabrication and construction constraints.

Material Properties and Joint Conditions

Timber and specifically plywood has been selected as the preferred study material for this research for a number of reasons. Firstly, timber is sustainable as a construction material as it can be grown again and is relatively inexpensive compared to other building materials. Secondly, although historically perceived as a liability for a

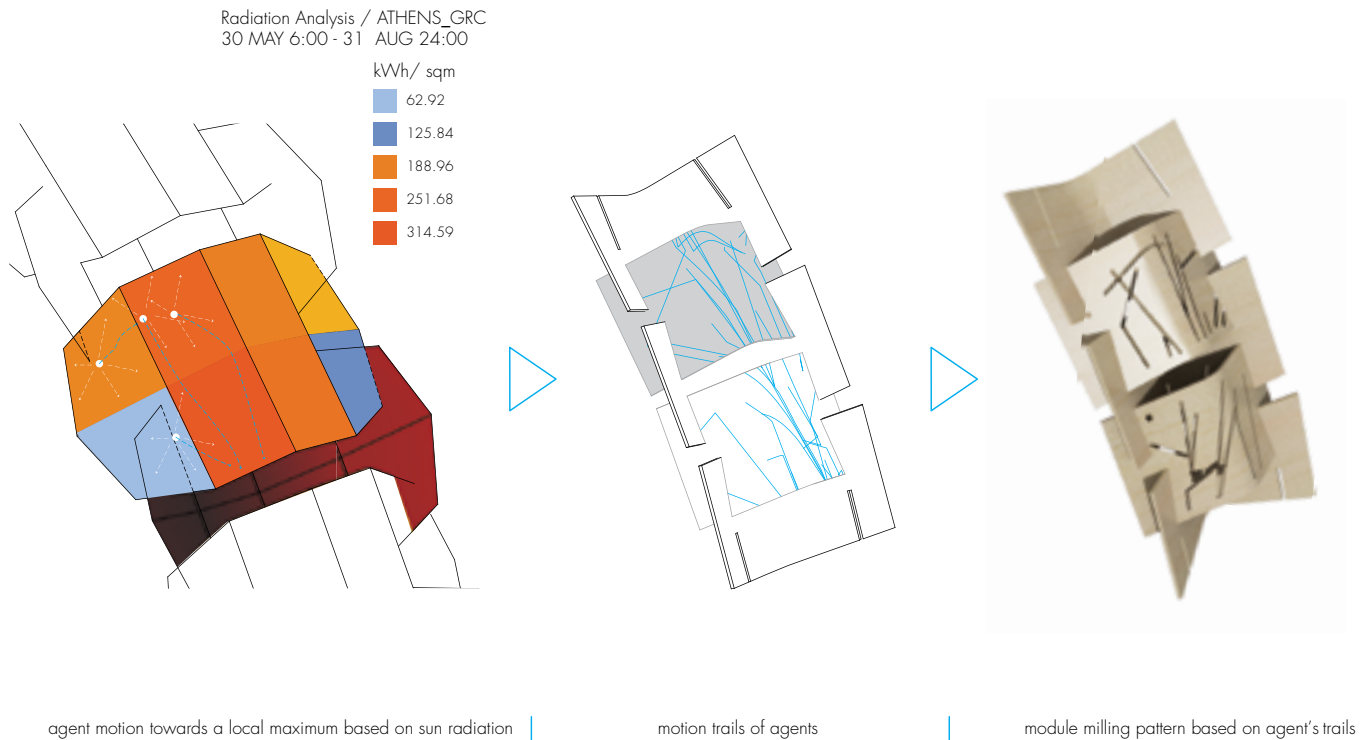


Figure 6: Diagram of the agent-based perforation system-behavior description and geometric translation

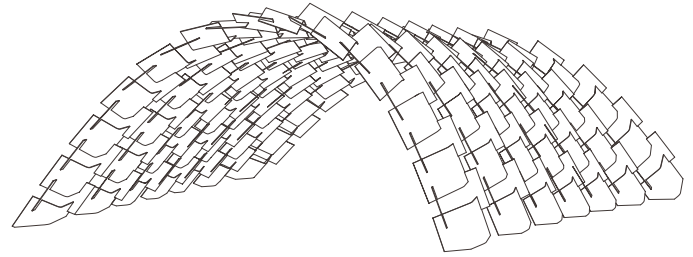
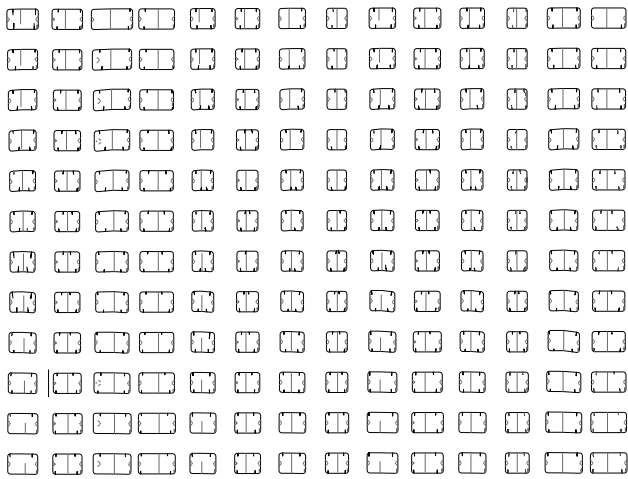
construction material, timber's fibrous nature can be considered an advantage in terms of both the material's functional and aesthetic characteristics. Additionally, in reciprocal structures components may vary in size and plywood offers the flexibility to be processed into different shapes and lengths through our digital fabrication workflows.

The design and detailing of the components' connections are intrinsic to each frame member whereby the whole system and consequently the mechanical behavior of joints, in terms of degrees of freedom, friction, deformability and displacement capability directly contribute to define the global behavior of the structure (Balfroid et al., 2011). The connection points become an integral part of the building component, and their complex and or unique geometry can be milled or directly fabricated into the volume (See Figure 3). The wood joinery tasks, historically manual and tedious, are now facilitated by means of advanced CNC facilities. Of interest is our ability to produce performance driven complex forms that can emerge from the aggregation of a basic module type by introducing notches with variable size and angle within the topology of the reciprocal frame.

Research Method

The methodology of the research has been to work in an incremental fashion with the overall objective of being able to measure improvements

in design outcomes in formal terms, in design performance and optimization terms, and finally in terms of a file to factory process affordances, all the while working with real world site, material, assembly, and cost constraints. For proving such an approach, a case study of a self-standing shell structure at 1:1 scale is realized and is sited on the rooftop of a cultural center in the center of Athens, Greece. (See Figure 4) As stated previously one aim is to develop computational framework for designing deployable structures that are informed by environmental data and that adapt to economical and material restrictions that also allow for local, vernacular stylistic propensities to be discussed in future research. A design to production workflow was developed that proceeds through the following steps: 1), form-finding of a canopy shell surface of limited area with two support conditions through the use of a mesh relaxation algorithm; 2) discretization of the generated surface through iso-curves that are populated with interlocking building components, where each component follows the principle of reciprocity and is modeled with an associative parametric geometry modeler; 3), informing the geometry of the basic element based on the selected material (thermo formed plywood) to optimize its transversal section and render it resistant to the forces under which the structure would be placed; 4), perform environmental analysis specifically solar radiation, and use this data for informing the probabilistic of a



54 sqm of plywood / 148 unique panels

24 sqm covered area

Figure 8: Diagram with the panels developed flat and assembled in the final structure

of mutually supported panels. The proposed slide connection scheme inspires a new family of reciprocal frames, where instead of linear members (beam or bar), folded or formed surface elements are mutually supported. The connection between the building components is integrated as a notch with specified angle within the geometry of members, unlike the traditional reciprocal frame system where the connector members are regular. A V-form module of given angle is fabricated through the process of thermoforming plywood panels and is then spatially multiplied using consecutive rotations and translations that follow the tangent vectors of a discretized catenary arch. The structure can be decomposed into three principal module types each consisting of curved panels (convex and concave) with locally specific angles in relationship to their generatively form found neighbors. These modules are then interlocked sequentially along their uniquely milled U shape cuts, to form an arch. The inter-panel stability is provided by rigidity of slide connection and axial contact of reciprocal panels. The structural performance of the whole structure improves when more than 2 arches are connected together. This is due to the fact that they act like a truss, with only axial compressive and tensile forces. Bending moments and shear forces minimize when in network arches (Tveit, 1987).

A Multi-Agent System for Material Fabrication

The workflow has been built on a series of open source simulation environments, an associative parametric design and visual programming environment Grasshopper/Rhinoceros and a series of plug-ins for integrating performance simulations and generating data that can then be read by a custom agent based swarm algorithm written in the Processing 2.0 environment. The specific Grasshopper definition incorporates Kangaroo, a physics simulation solver used for running a mesh relaxation algorithm on a shell structure with given boundary conditions. The generated surfaces are parametrically discretized in iso-curves, which are further divided in linear segments. Interlocking components are placed sequentially in pairs on the division points of each iso-curve and are oriented parallel to the tangent vectors. The panels are further analysed with Ladybug, another Grasshopper plugin that runs a Radiance based simulation. All components are numbered and unrolled in flat panels with all the perforation and notch lines projected. Thus material calculations can be done relating the covered area of the shell surface with The simulation data is exported as a text file that is passed to the custom flocking algorithm in the Processing environment in 3D where the agents read the vertices of the component surfaces and coupled data values. The agents are spawned and programmed to make movement



Figure 9: Photo of the assembled pavilion

and trajectory decisions based on the local information including the intensity values from the sun simulation, proximity to neighbors, and trails left by other agents. Each agent has the capacity to read the data from the simulation, which is paired with its corresponding point in a mesh object as well as data related to its neighbors' and constraints as trajectories in time. Thus, the agent's environment is a collection of points to which it is constrained, and each point is assigned an intensity score based on the data from the simulation. The agent's trajectories then become a generative geometry for material organization which happens in a collective recursion. The agent-based trails are exported again as a text file into the Rhinoceros / Grasshopper scripts to be incorporate as another layer of information that controls the permeability of the panels through a process of CNC driven material subtraction through milling. (See Figure 7)

Fabrication process and 1:1 Prototyping

In order to test the structural feasibility of such a design approach a prototype at 1 to 1 scale was produced. A selection of photos illustrating the project is shown in (See Figure 9,10). The design and programming of the structure was at first optimized for discretizing an irregular surface in curved panels using a single pressing mold. The size of the structure was parameterized based

on a maximum amount (area) of material, that of 54 (See Figure 8) sqm. The whole production workflow included; 1), material processing; 2), manufacturing of the pressing mold; 3), CNC cutting of the components; 4), thermoforming and post processing of the components; and 5), final assembly and erection on site.

Of great significance for material efficiency was the creation of plywood panels in sizes that corresponded to the available veneer sizes and not the standard dimensions. This saved material as plies were used "as is" and reduced material processing time as multiple plies needed not to be stitched together in larger panels before being glued to form the final plywood panel that was CNC milled. Moreover the module's geometry was optimized in order to be fabricated by a single mold for reducing production costs. The panels were thermo-glued at first flat and were milled afterwards using a 5-axis CNC machine. The cut pieces were then reheated and pressed consecutively in the mold to take the final shape. A master parametric 3d model was developed in Grasshopper-Rhinoceros that generated all the cutting files for the 148 components in a file-to-factory process where custom routines were developed to give each component their precisely calculated slots for the sliding joints, all in gradually shifting positions and variable angles in order for the pavilion to achieve its irregular funicular form. All components were uniquely



Figure 10: Detail photos of the structure

labeled and numbered in order to facilitate assembling and dismantling of the structure without the need of expert workers or detailed drawings.

The plywood components were manufactured in Greece at a factory outside Athens and transported to the site. On site assembly of the pavilion was completed in two days, by a group of 5 non expert workers.

Material behavior and performance

Through our approach to the reciprocal frame we investigated the potential of using curved plywood components instead of planar ones. For that reason we selected an innovative material called UPM Grada, which is specifically designed for the manufacturing of form pressed plywood panels. UPM Grada uses the application of an adhesive film which allows the plywood to be thermo-formed after production by applying pressure through a custom made mold. The UPM Grada technology allows for the fabrication of custom panels as the film can be cut according to the available veneers for material. It was empirically discovered that the film adhesive and thermo-forming process produced panels with more elastic behavior than the traditional gluing technique which uses phenol-formaldehyde based wood glue and high voltage electricity for curing the glue while the panel is being pressed.

This facilitated the assembly process as it allowed the panels to slide in place more easily. Given the common cost and material constraints locally but arguably globally for projects like these, the project sought to be highly resourceful with material usage aided by computational design tools and CNC manufacturing. The thin self-standing shell structure with a footprint of 24 sqm required 54 sqm (0.4 cubic meters) of thermo-formed birch plywood. The total scrap wood did not exceed 5 sqm and was used to create the seating below the structure.

In further comparison to the precedent work (Weinand, 2011) we have attempted to define benchmarks and measures for the affordances of our approach. By adapting the principle of reciprocity and a specific family of reciprocal frames with mutually supported curved panels we benchmarked the construction of a maximum 600 cm span out of 45 cm long, 7.5mm thick interlocking panels that require no additional joinery. Previous structure, following the same principle, realized by prof. Y. Weinand, S. Nabaei and students in Lausanne had achieved a span of 740cm with 21 mm thick panels (Nabaei and Weinand, 2011). While the project is clearly indicative and statistically descriptive of improved span to material depth we expect to continue to investigate the project for further efficiencies of span, material usage, and applicability to complex curvature and other case studies. What is also a major thrust of our future investigation is that

of the impact of the swarm generated pattering and its influence on the structural as well as environmental performance of the generated geometries

Conclusion and discussion

The paper presents our very first built experiment which combines research into the development of a workflow that combines the possibilities of structural form finding techniques with bottom up strategies that implement materially and environmentally informed swarm agents. The canopy also, from inception through to its use, incorporated intuited and then empirical real world material, assembly, cost, and human constraints that informed the development of the system. While we have yet to measure the results of all of our research objectives, those of material optimization, multi-agent based design objectives, structural performance and fabrication times and efficiencies are the most concrete so far. As stated we will continue to investigate these further through more empiricism but as well will further develop the use of the multi-agent system for its articulation possibilities and for their ability to collectively negotiate even more optimal material and environmental conditions for the reciprocal frame systems. Reconfigurable structures and resource saving, mutually supported building systems like reciprocal frames will be further investigated in concert with digital form finding and articulation. Clearly we can observe that if the internal forces are known, each element in such structure can be adjusted to its local stress and therefore an optimized material consumption can be achieved. It is through computation and digital fabrication processes that such approaches become easily realizable and economically viable.

This work is part of an ongoing research into multi-objective design systems that seek to incorporate a bottom up MAS approach for architecture through the combination of form finding techniques with agent based design systems and digital fabrication processes. The aim of this work was to develop a workflow combined with a fabrication protocol that has the potential to bridge design intentions with a structural system and realistic parameters (budget) with material constraints. The focus here was to take advantage of intrinsic features of the material properties and assembly methods and apply simple local rules in a manner that

enables emergent phenomena to arise. In such an approach the design intention is codified by these local and global constraints and their relations rather than by a prevailing “architectural gesture.” The design optimization becomes an iterative process, where both solution and starting condition are constantly oscillating towards an equilibrium defined by multiple performance criteria in response to the generated geometry, the component’s geometry and the user’s design intentions and adjustments. This study presents an initial analysis of the generative and automation possibilities using a digital fabrication workflow driven by a simulated and recursively informed material swarm aggregation for a novel reciprocal frame design approach.

Acknowledgements

We would like to thank and acknowledge Xenakis Curved Plywood S.A, Romantso Cultural Center, Ikea Stiftung in Switzerland and Onassis Foundation for their support of the project and of the authors; and to the team of helpers Iason Pantazis, Constantinos Schinas, Anastasios Spyridwnos, Dimitris Charitatos, Thanasis Demiris, Iraklis Kassimis, Dionysis Dikefalos, Viktoras Gogas, Anastasios Dedes and Rodrigo Shiordia for their tireless support and contributions. This material is in part based upon work supported by the National Science Foundation under Grant No. 1231001. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

References

- BAHARLOU, E. & MENGES, A. 2013. Generative Agent-Based Design Computation.
- BALFROID, N., KIRKEGAARD, P. H. & SASSONE, M. S. Robustness of Long Span Reciprocal Timber Structures. The 35th Annual Symposium of the IABSE 2011, the 52nd Annual Symposium of the IASS 2011 and incorporating the 6th International Conference on Space Structures, 2011 London, United Kingdom.
- GERBER, D. J. 2007. Parametric practices: Models for design exploration in architecture. D.Des. Dissertation, Harvard Graduate School of Design.
- GERBER, D. J. & LIN, S.-H. E. 2013. Designing in complexity: Simulation, integration, and multidisciplinary design optimization for architecture. Simulation, Published online before print April 9, 2013.
- GILBERT, N. 2008. Agent-based models, Sage.
- HENSEL, M., MENGES, A. & WEINSTOCK, M. 2010. Emergent technologies and design, Oxon, [U.K.] ;New York, NY, Routledge.
- IRELAND, T. 2009. Emergent space diagrams: the application of swarm intelligence to the problem of automatic plan generation. Joining Languages, Cultures and Visions: Proceedings of the 13th International CAAD Futures Conference. Montréal, Canada: Les Presses de l'Université de Montréal.
- KILIAN, A. & OCHSENDORF, J. 2005. Particle-spring systems for structural form finding. JOURNAL-INTERNATIONAL ASSOCIATION FOR SHELL AND SPATIAL STRUCTURES, 148, 77.
- KOHLHAMMER, T. & KOTNIK, T. 2011. Systemic behaviour of plane reciprocal frame structures. Structural Engineering International, 21, 80-86.
- KWINTER, S. & DAVIDSON, C. 2008. Far from equilibrium: essays on technology and design culture, ACTA Press.
- LARSEN, O. P. 2008. Reciprocal frame architecture, Routledge.
- LIN, S.-H. E. & GERBER, D. J. 2014. Designing-in performance: A framework for evolutionary energy performance feedback in early stage design. Automation in Construction, 38, 59-73.
- MIRANDA, P. & COATES, P. 2000. Swarm modelling. the use of Swarm Intelligence to generate architectural form. Proceedings of the 3rd Generative Art Conference. Milan, Italy: AleaDesign Publisher.
- MOREL, P. 2006. Computational Intelligence: The Grid as Post-Human Network. AD. John Wiley & Sons Ltd. .
- NABAEI, S. S. & WEINAND, Y. 2011. Geometrical description and structural analysis of a modular timber structure. International Journal of Space Structures, 26, 321-330.
- PIZZIGONI, A. 2010. LEONARDO & THE RECIPROCAL STRUCTURES.
- PUGNALE, A., PARIGI, D., KIRKEGAARD, P. H. & SASSONE, M. S. The principle of structural reciprocity: history, properties and design issues. The 35th Annual Symposium of the IABSE 2011, the 52nd Annual Symposium of the IASS 2011 and incorporating the 6th International Conference on Space Structures, 2011.
- REYNOLDS, C. W. 1987. Flocks, herds and schools: A distributed behavioral model. ACM SIGGRAPH Computer Graphics, 21, 25-34.
- SCHEURER, F. 2005. Turning the design process downside-up. Computer Aided Architectural Design Futures 2005. Springer.
- SCHEURER, F., SCHINDLER, C. & BRAACH, M. From design to production: Three complex structures materialised in wood. 6th International Conference Generative Art, 2005.
- TSILIAKOS, M. 2012. Swarm Materiality: A multi-agent approach to stress driven material organization. Digital Physicality: Proceedings of the 30th eCAADe Conference. Prague, Czech Republic: Czech Technical University in Prague, Faculty of Architecture.
- TVEIT, P. 1987. Considerations for Design of Network Arches. Journal of Structural Engineering, 113, 2189-2207.
- VAN MELE, T., DE LAET, L., VEENENDAAL, D., MOLLAERT, M. & BLOCK, P. 2013. Shaping Tension Structures with Actively Bent Linear Elements. International Journal of Space Structures, 28, 127-136.
- WEINAND, Y. 2011. Innovative Timber Constructions. IABSE-IASS 2011. London, Great-Britain.